[redacted]

5-Person Team Test

Abbreviated Instructions: Answer each of the following questions **using separate sheet(s) of paper for each numbered problem**.

- Place your team code in the upper right corner of each page that will be turned in.
- Place problem numbers in the upper left corner (failure to do these things will result in no score for that problem/page).

Problems are equally weighted; **teams must show complete solutions (not just answers) to receive credit**. More specific instructions are read verbally at the beginning of the test.

1. Given a square of unit area:

   a. Show that the square can be partitioned into six squares (Note: The squares do NOT [redacted]

4. A circle has both an inscribed and circumscribed regular polygon (both having the same number of sides). Find the ratio of areas for the larger polygon to the smaller:

   a. If the polygon is a triangle.
   b. If the polygon is a square.
   c. If the polygon is a hexagon.
   d. If the polygon is has n sides. As n gets large, what number does the ratio approach?

5. Jayden and Cody decide to play a coin flipping game. They decide to flip a fair coin until they obtain a sequence of either five consecutive heads or five consecutive tails, at which point the game will end.

   a. What is the probability the game ends within the first five flips?
   b. What is the probability the game ends within the first six flips?
   c. What is the probability the game ends within the first seven flips?

Problem #2

2 Le[illegible] be the produc[illegible] [illegible]le[illegible]nts and s be the sum of elts in P. Then since [illegible]he sum [illegible]f all ten numbers [illegible] $\frac{10(11)}{2} = 55$, we have tha[illegible] $p = 55 -$ [illegible] n [illegible] $s = 5$ for numbers [illegible] [illegible] sum [illegible] [illegible]

Note t[illegible] $\cdot 2 \cdot 3 \cdot 4 \cdot$ [illegible] > 5 [illegible] so [illegible] product [illegible] satisfies [illegible] will have more than [illegible] numbers. [illegible]lso there canno[illegible] be only one number [illegible] [illegible] $x = 55$ has [illegible] so [illegible] ≤ 1.

(The solutions are $P = \{6, 7\}$, $P = \{1, 4, 10\}$ and $P = \{1, 2, 3, 7\}$

[illegible]

$x = 1$ [illegible] $) =$ [illegible]

[illegible]

$= 2$ [illegible] 2[illegible] $+ (2 + 4) = 55 \Rightarrow 3y = 53$ " "

$3^{\text{etc}} \Rightarrow$ do $x - 5$ no sol'n.

[illegible] x [illegible] $+ (x + y$ [illegible] $z = 55$

[illegible] y, z all odd [illegible] is also odd.

[illegible] 4 odds ca[illegible] sum to 55.

[illegible] one elt, say x, is o[illegible] [illegible] ven n s $xyz + (x + \quad z) = 55$ is on)

possible if x ...) u ...

one odd

... 2 even

$2 \cdot 4 \cdot z + (2 + 4 +$ $= 5.$ $9z = 49$ No Sol

1 2 $= 55$ $3 \quad = 4$ " "

1 $+$ $2 + ^0 +$ $=$ $17 = 45$ " "

$2 \cdot 10 \, z + 2 + 10 +$ $= 55$ $21z = 43$ " "

$\cdot 6 + (+ 6 +$ $= 55$ $25z = 45$ " "

$4 \cdot 8 \, z + (4 + 8 + \;) = 55$ $33z = 43$ " "

* $\cdot 10$ $+10 \cdot z) = 55$ $41z = 41$ $z = 1$

[illegible]

$\cdot 10$ $+ 2$ $61z =$ " "

on 3 ... $(4 \quad 0)$

4ck $= (x,$ 8 w $x,$ deg lan $>$

$1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 + (1 + 2 + 4 +$ $= 54$ $2 + \; 5$ No

$1 \cdot 2 \cdot 4$ $+$ $+ 3 + 5) = 55$ $30 + 11 \quad 55$ No

$\cdot 2 \cdot 3 \cdot$ $+ (1 + 2 + 3 + 6) = 55$ N

$1 \cdot 2 \cdot 3 \cdot 5 + (1 + 2 + 3 + 5) =$ $42 + 13 = 55$ Yes

$1 \cdot 2 \cdot 3 \cdot x$ Anything $x >$ $\cdot 11$ e e! No

its end $1, 2, 3$ $) =$

Problem #3

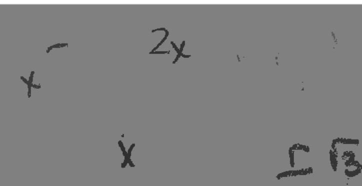
3) (1, 2, 3, 4 +, − * arentheses

a) Max: 19 = 4*(3+2) − 1

b) Min: −19 = 1 − 4*(3+2

c) s : +2−3 *4 = 0

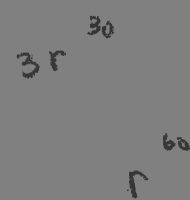
Problem #4



Small Δ A = 6 b = 6 1/2 ($\frac{3r}{2}$ $\frac{}{2}$

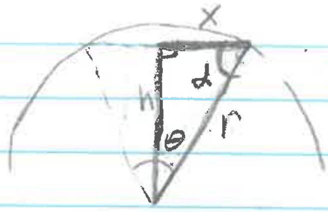
=

$A = 6 \cdot \frac{1}{2} bh = 6\left(\frac{1}{2}\sqrt{3}r)\, r\right) = 3\sqrt{3}r$



r larc - $\frac{3\,3\,r^2}{2}$ -

general



Small n-gon $A = \frac{1}{\ } x \cdot \ = \frac{1}{2} \ r \sin\ ) \ r\cos\alpha = \frac{\ }{\ } r^2 \sin\alpha\cos\alpha$

$\sin\alpha = \frac{h}{r}$ $\quad\quad$ $\cos \ = \frac{x}{r}$

α $\quad\quad$ $= r \cdot \cos\alpha$

x

Lar n- A $x \cdot r = \frac{1}{\ } \ \frac{r}{\ } \ r$

$=$ $\quad\quad$ $= \ 2$

x

$x =$

a

2

=

Sin $\frac{2\tan\alpha}{\ } = 2\sin\alpha \ \sin\alpha\cos\alpha$

$= \frac{1}{\sin}$ -

Problem #5

5 5 Heads on a ls in a row

a 5H v 5 t of 5 rolls

$\frac{}{2} + \frac{}{2} = \frac{1}{2}) = 1$

or

$1 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{} = 1/$

any same same same same

b) Prob oct of = Pr (1st 5 st 5

$\cdot \left(\frac{1}{2}\right)^6 = \frac{1}{16} + \frac{1}{} = 3/32$

HTTTTT or THHHHH

from

3/ + 4 · 1

HT 5H
HH 5T
TH 5T
TT 5T

d) If we re eat part th nob likes of 1/3 a 2/3, we t

$\frac{5}{3} \quad 2 \quad 5 = \frac{1+32}{243} \quad \frac{33}{243} > 26$

So m r likely end e in

Problem #6

) $y^2 = x^2 + b$ c be wn #rn

$- \; ^2 \quad y^2 - \; ^2 = y - x) \quad + \quad =$

a $= 24$ $(y-x)(y+ \quad = 2$

-actor pairs for 24: (1·24 (2·12 3·) (4·6

$^2 , 12 \quad = \quad =$ The o ler no

(4·6 $= 5$ $= 1$ inte n 1 ecus.

b) $b = 60$ $(y-x)(+x = 60$

·6 2·30 / 3·20 / 4·15 / 5·12 6·1

2·30 $= 16$ $x = 1$ diffrenc for

6·10 $=$ $x =$ the o h r factor s.

c $= 210$ Hr 21 $= 2 \cdot 3 \cdot 5 \cdot 7$

no matter ow you factor, in dh a

1 be ven & one wil e dd & so, as

above, there are no solutions